Journal of Engineering Physics and Thermophysics, Vol. 77, No. 4, 2004

HEAT CONDUCTION AND DIFFUSION EQUATION OF STEAM IN SNOW COVER

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UDC 551.578.46:551.57

Usually, the temperature gradient in snow cover is many times over the critical value above which convective motion of air confined in snow pores begins. These motions strongly affect the rate of heat and mass transfer processes in the snow cover. Analytical formulas that express the dependence of the generalized heat-conduction and diffusion equations of steam in snow on the Peclet number are obtained. A reason has been elucidated due to which, in most cases, the value of the diffusion coefficient of steam in snow cover is higher than its value in air.

The heat-conduction and diffusion equations of steam are the most important parameters that determine the thermophysical properties of snow; therefore, many works have been devoted to experimental determination (in both laboratory and field conditions) of these coefficients [1, 2]. The obtained values of the coefficients are characterized by rather wide scatter and depend on the type of snow, its density and granular structure, on the experimental conditions, etc. The diffusion coefficient of steam has the largest scatter; the measured values of it lie within the limits (0.13–1.1)$\cdot 10^{-4}$ m^2/sec [2] and in most cases they exceed the value of the diffusion coefficient of steam in air (~0.2$\cdot 10^{-4}$ m^2/sec). High values of the diffusion coefficient of steam in snow indicate the presence of intense moisture flow within the thickness of the caked snow, which lead to substantial changes in the structure and thermophysical characteristics of snow.

In the present paper, we try to unravel the mechanism of origination of these flows. It follows from the theoretical studies [3] that the air in the pores loses stability and convection begins at temperature gradients in the snow cover that exceed some critical value γ_{cr}, determined as

$$\gamma_{cr} = \frac{\nu \chi_s R'_{a,\min}}{\beta f g M H^2 \sigma}, \tag{1}$$

where $\beta = 1/T$, M = 1.8–2.2, and, $R'_{a,\min} \approx 30$ is the Rayleigh number for snow at which convective motion of air in the snow cover begins.

To determine the permeability coefficient of snow we use the semi-empirical Kozeny formula [4]

$$\sigma = \frac{f^3}{150\,(1-f)^2}\,d^2 = \frac{\left(1+\dfrac{\rho_s}{\rho_i}\right)^3}{150\left(\dfrac{\rho_s}{\rho_i}\right)^2}\,d^2 .$$

At typical values of the parameters $\chi_s = 4\cdot 10^{-7}$ m^2/sec, $\sigma = 3.6\cdot 10^{-8}$ m^2 (d = 0.2 cm), f = 0.6, T = 260 K, M = 2, $\nu = 0.15\cdot 10^{-4}$ m^2/sec, and H = 0.5 m, formula (1) yields $\gamma_{cr} \approx 0.5$ deg/m. By the data of [3] real temperature gradients greatly exceed γ_{cr}. Hence it follows that at a rather large thickness of the snow cover, convective cells with

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a linear size of about $2\pi H/k_{max}$ ($k_{max} \approx 2.3$–2.5 is the maximum value of the dimensionless wave number for the basic level of air instability in the snow cover) appear in it.

Steady-state vertical air flows in the snow cover are described by the equation

$$V\frac{dV}{dz} = \beta g H\,(\gamma - \gamma_{cr}) - \frac{\nu}{\sigma}V\,, \tag{2}$$

where the z axis is directed vertically upward from the lower boundary of the snow cover.

Analysis of the solution of (2) which satisfies the condition $V(0) = 0$ shows that velocity V reaches a maximum value $V_{max} = \frac{\beta g \sigma H}{\nu}(\gamma - \gamma_{cr})$ at a distance of an order of several centimeters from the lower boundary of the snow cover; then air ascends at constant velocity. Thus, we can approximately assume that at a certain distance from the lower boundary the rate of filtration within the entire snow thickness is constant and equals

$$u = fV = \frac{f\beta g\sigma H}{\nu}\,(\gamma - \gamma_{cr})\,. \tag{3}$$

We can now present the equation of convective heat conduction in the snow cover as [3]

$$\rho_s c_s \frac{\partial\theta}{\partial t} = \lambda_s \Delta\theta - \mathrm{div}\,(\rho c_p \mathbf{u}\theta) - L\,\mathrm{div}\,(\rho_w \mathbf{u}) + LD\Delta\rho_w\,. \tag{4}$$

The third and fourth terms on the right-hand side of Eq. (4) describe the contribution of steam condensation of ice crystals due to a decrease in temperature of the air mass ascending in the snow.

Proceeding from the Clapeyron–Clausius equation, the relation between ρ_w and θ can approximately be written as

$$\nabla\rho_w = \frac{L\rho_{w0}}{R_w T_0^2}\nabla\theta \tag{5}$$

(∇ is the gradient operator and $T_0 = 273$ K).

With account for (5), in the steady state the equation of heat conduction in the snow cover has the form

$$\frac{\rho c_p}{\rho_s c_s}\,uM\frac{\partial\theta}{\partial z} = \chi_s\frac{\partial^2\theta}{\partial z^2}\,, \tag{6}$$

where

$$M = 1 + \frac{L^2\rho_{w0}}{R_w T_0^2 \rho c_p}\,; \quad \chi_s = \frac{\lambda_s}{\rho_s c_s}\left(1 + \frac{\rho_{w0} L^2 D}{R_w \lambda_s T_0^2}\right).$$

If we denote the temperatures on the lower and upper boundaries of the snow cover by θ_1 and θ_2, solution of Eq. (6) takes on the following form:

$$\theta\,(z) = \frac{\theta_1 \exp a - \theta_2 - (\theta_1 - \theta_2)\exp\left(a\frac{z}{H}\right)}{\exp a - 1}\,,$$

here

$$a = \frac{\rho c_p}{\rho_s c_s} \frac{f \beta g \sigma H M}{\chi_s u}$$

is the Peclet number. Hence we obtain

$$\frac{d\theta}{dz} = -\frac{\Delta\theta}{H} \frac{a \exp\left(a \frac{z}{H}\right)}{\exp a - 1}, \tag{7}$$

where $\Delta\theta = \theta_1 - \theta_2$. In Eq. (7), the ratio $\Delta\theta/H$ is the temperature gradient in the absence of convection in the snow cover; thus, the multiplier at it shows by how many times the intensity of heat transfer due to air convection increases compared to the same intensity due to molecular heat conduction. Thus, we can write

$$\lambda_{\text{eff}} = \lambda_s \frac{a \exp\left(\frac{az}{H}\right)}{\exp a - 1}. \tag{8}$$

At typical values of the parameters in (1) and an air density of $\rho = 1.29$ kg/m^3, we obtain $u = 5.54 \cdot 10^{-5} \Delta\theta$ m/sec and $a = 0.45\Delta\theta$ for the rate of filtration and the Peclet number, respectively. Hence it follows that in the range $\Delta\theta \approx (5\text{–}10)^{\text{o}}$C the effective heat-conduction equation of the snow cover changes from λ_s to $a\lambda_s$.

In the steady-state processes, the equation of mass transfer in the snow cover takes on the form

$$u \frac{d\rho_w}{dz} = D_s \frac{d^2 \rho_w}{dz^2}. \tag{9}$$

The relation between the coefficient D_s and the coefficient of diffusion of steam in air D can be found from the following considerations. The volume of snow is pierced by multiple microchannels through which steam is seeping. The relative cross-section area of these microchannels is

$$S = \delta^2 n^{2/3}. \tag{10}$$

On the other hand, since the coefficient of porosity of snow is expressed in terms of n and δ by the formula

$$f = n\delta^3, \tag{11}$$

from equalities (10) and (11) we have $S = f^{2/3}$.

Thus, the relative area of the cross section through which steam can diffuse is of about $f^{2/3}$ and, consequently,

$$D_s \approx f^{2/3} D. \tag{12}$$

Solution of Eq. (9), which satisfies the boundary conditions

$$\rho_w \big|_{z=0} = \rho_w(\theta_1), \quad \rho_w \big|_{z=H} = \rho_w(\theta_2),$$

has a form similar to solution of Eq. (6). Then we obtain

$$\frac{d\rho_w}{dz} = -\frac{\rho_w(\theta_1) - \rho_w(\theta_2)}{H} \frac{a_1 \exp\left(a_1 \frac{z}{H}\right)}{\exp a_1 - 1}, \tag{13}$$

where

$$a_1 = \frac{uH}{D_s} = \frac{\beta g H f \sigma}{\nu D f^{2/3}} \Delta\theta .$$

Hence, as earlier, we have for the effective coefficient of diffusion

$$D_{eff} = D_s \frac{a_1 \exp\left(a_1 \frac{z}{H}\right)}{\exp a_1 - 1} . \tag{14}$$

At typical values of the parameters that are involved in (13), a_1 = 1.95$\Delta\theta$; at H = 0.5 m and $\gamma - \gamma_{cr}$ = 10 deg/m, $a_1 \approx 10$, which is 1.7-fold larger than a. It is seen from these estimates that convective motion of air in the snow cover exerts a stronger effect on the processes of moisture transfer than on the processes of heat transfer.

Calculations show that at a pore diameter in the snow of d = 0.2, 0.15, and 0.1 cm the effective coefficient of diffusion D_{eff} is $1.4 \cdot 10^{-4}$, $0.78 \cdot 10^{-4}$, and $0.35 \cdot 10^{-4}$ m^2/sec, respectively. These values are in good agreement with the experimental data of different authors that are given, e.g., in [1, 2]. Moreover, when $a_1 > 2$ the coefficient D_{eff} is proportional to the temperature gradient within the snow thickness, which is also in agreement with the experimental data of [1].

Thus, we can take it to be ascertained that the high values of the coefficient of diffusion of steam in the snow cover compared to its value in air are stipulated by the existence of convective motion of air within the snow thickness.

It follows from formulas (8) and (14) that as the Peclet numbers a and a_1 increase, the gradients of temperature and humidity increase, going toward the boundary of the snow cover. Consequently, the processes of heat and mass transfer in the presence of convection are more intense at this boundary.

It also follows from the calculations that, along with the coefficients of heat conduction and diffusion, the coefficient of snow permeability is the most important parameter that determines the processes of heat and moisture transfer in the snow cover. Therefore, one of the problems of the physics of snow is to develop prompt methods of determining it.

NOTATION

a, Peclet number; a_1, Peclet number for mass transfer; c_p, heat capacity of air at constant pressure, J/(kg·deg); c_s, heat capacity of snow, J/(kg·deg); D, coefficient of steam diffusion in air, m^2/sec; D_s, coefficient of steam diffusion in snow in the absence of convection, m^2/sec; D_{eff}, effective coefficient of steam diffusion in snow, m^2/sec; d, mean diameter of pores in snow, m; f, coefficient of snow porosity; g, free-fall acceleration, m^2/sec; H, snow-cover thickness, m; k_{max}, maximum value of the dimensionless wave number, m^{-1}; L, specific heat of steam sublimation, J/kg; M, numerical coefficient; n, number of pores per snow-volume unit, m^{-3}; R_w, gas constant of steam, J/(kg·deg); $R'_{a,min}$, minimum Rayleigh number for snow; S, relative area of pores in snow; T, absolute temperature; t, time, sec; u and $\mathbf{u}$, rate of filtration and its vector, m/sec; V, vertical component of air velocity in snow, m/sec; β, coefficient of thermal expansion of air, deg^{-1}; γ, temperature gradient in snow, deg/m; γ_{cr}, critical temperature gradient at which air confined within the snow pores loses its stability, deg/m; δ, mean diameter of pores in snow, m; θ_s, temperature in the snow cover, ^{o}C; θ_1 and θ_2, temperatures of the lower and upper boundaries of the snow cover, ^{o}C; χ_s, coefficient of thermal diffusivity of snow in the absence of convection, m^2/sec; λ_s, coefficient of thermal conductivity of snow in the absence of convection, J/(m·sec·deg); λ_{eff}, effective coefficient of thermal conductivity of snow, J/(m·sec·deg); ν, kinematic coefficient of viscosity of air, m^2/sec; ρ, air density, kg/m^3; ρ_s and ρ_i, density of snow and ice, respectively; kg/m^3; ρ_w, saturating density of steam over a plane surface of ice at a temperature of θ_s, kg/m^3; ρ_{w0}, saturating density of steam over the ice surface at 0^{o}C, kg/m^3; σ, coefficient of snow permeability. Subscripts: s, snow; p, pressure; eff, effective; max, maximum; min, minimum; cr, critical; w, steam; a, air, i, ice.

REFERENCES

1. V. I. Fedoseeva and N. F. Fedoseev, Evaluation of the coefficient of diffusion of water vapor in snow cover, *Meteorol. Gidrol.*, No. 2, 132–135 (1988).
2. M. A. Dolov and V. A. Khalkechev, *Physics of Snow and Dynamics of Snow Avalanches* [in Russian], Gidrometeoizdat, Leningrad (1972).
3. M. K. Zhekhamukhov and I. M. Zhekhamukhova, Convective instability of air in snow cover, *Inzh.-Fiz. Zh.*, **75**, No. 4, 65–72 (2002).
4. G. Z. Gershuni and E. M. Zhukhovitskii, *Convective Stability of Incompressible Liquid* [in Russian], Nauka, Moscow (1972).